



# Servo Loop Bandwidth, Motor Sizing and Power Dissipation

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## HOW SERVO LOOP BANDWIDTH, MOTOR SIZING AND POWER DISSIPATION ARE ALL RELATED

This paper will describe how servo loop bandwidth, motor sizing and power dissipation are all related. Some background regarding servo loop gains and bandwidths will be presented to set the stage for a discussion of two case studies. Case 1 is a move and settle application where it will be shown that servo bandwidth is not related to motor sizing. Case 2 is a disturbance tracking application where servo bandwidth directly affects motor sizing. In each case, power dissipation is ultimately the primary driver for sizing a motor.

## LOOP GAIN, STABILITY, BANDWIDTH AND MOTOR SIZING

Servo bandwidth is a direct function of loop gain, which includes gains from the plant, the sensor, the motor, the motor drive and the control gain. Higher loop gain always results in higher bandwidth. Loop gain can be increased or decreased simply by adjusting the gain of any one component in the loop (control, plant, sensor, motor or motor drive). This means that there is not a single solution of each gain that results in a targeted bandwidth, rather, each gain can be lower or higher, as long as the total gain is conserved. For example, the sensor gain for system A could be 1 count for 1  $\mu\text{m}$ , and the sensor gain for system B could be 10 counts for 1  $\mu\text{m}$ . System A and B can have the same bandwidth as long as one of the other gains makes up for the 10x difference. In this example, the most likely gain to adjust would be the control gain, system B being 10x less.

Power dissipation aside, loop gain (and resulting bandwidth) cannot grow without bound, rather it is limited by stability. Stability is driven by phase margin at the bandwidth frequency and gain margin when the phase is at -180 degrees.

Higher bandwidths mean the motor will be commanded by the control system to respond at higher frequencies. Higher frequencies result in higher accelerations and higher required motor forces, all resulting in higher dissipated power. Dissipated power (or allowable temp rise), can limit the allowable bandwidth, meaning that although stability might allow a system to have a 100 Hz bandwidth, the dissipated power requirement and motor  $K_m$  chosen might limit the bandwidth to be only 50 Hz. Or conversely, if 100 Hz is needed, the motor  $K_m$  will be sized accordingly to meet the power dissipation requirement.

## CASE 1- MOVE AND SETTLE

In this case it will be shown how motor sizing is not coupled to servo bandwidth, but is couple to power dissipated in the motor.

Figure 1 below is a generic block diagram that is valid for almost all closed loop control systems. As stated earlier, the loop gain is comprised of the control gain ( $K_{cntrl}$ ), the drive gain ( $K_{drive}$ ), the motor gain ( $K_t$ ), the plant gain ( $K_{plant}$ ) and the sensor gain ( $K_{sens}$ ).  $K_{cntrl}$ ,  $K_{drive}$  and  $K_{plant}$  are highly frequency dependent.  $K_t$ , and  $K_{sens}$ , however, are typically constant (up to a few hundred Hz). If we have a targeted bandwidth frequency, we can solve for  $K_{cntrl}$  at desired bandwidth frequency and confirm stability.

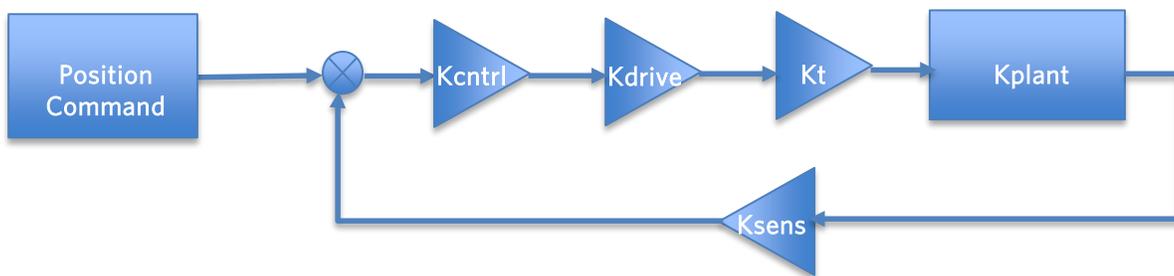


Figure 1 System Block Diagram

In the system shown in Figure 1, the plant is typically a fixed entity. For the sake of this example, let's say the sensor gain is also fixed. Now let's say system A had a motor gain of 10 N-m/amp and system B, 1 N-m/amp. As we described earlier, system A and B can both have the same bandwidth, provided the overall loop gain is the same. In this example, if  $K_{cntrl}$  system B were increased by 10x, system A and system B would have the same loop gain, and therefore the same servo bandwidth.

This example is intended to show that there is not a unique motor gain that provides a desired servo bandwidth. There is however, a unique torque required to move the load, given the inertia of the load and its acceleration. Changing the motor gain (N-m/amp) between system A and B means that to conserve the torque created by the motors, the current flowing through the motor for system B is 10x larger than that of system A. This also means that the dissipated power of system B will be larger than that of system A, and that system B will have a higher coil temperature.

When moving a load from one location to another, the servo bandwidth does not significantly impact the motion trajectory. Figure 2 below depicts how different servo bandwidths will create very similar motion

trajectories. Clearly the 100 Hz servo follows the commanded position better than the 50 Hz, but the dominant acceleration is created from the trajectory itself.

The acceleration, along with the duty cycle of the movement, and the mass or inertia of the load, all affect the dissipated power (Pd).

$$Pd = \frac{\text{Duty Cycle\%} * \text{Torque}^2}{K_m}$$

where Duty Cycle% is the  $\frac{\text{time accelerating}}{\text{time accelerating} + \text{dwell time}}$ ,

$K_m$  is  $\frac{N - m}{\sqrt{Watt}}$  and

Torque = inertia (N - m<sup>2</sup>) \* acceleration (rad/.s<sup>2</sup>).

Knowing an allowable Pd means you can solve for a  $K_m$  value and motor sizing is now complete. Motor current and voltage can be traded by the motor design team to arrive at the optimal  $K_t$ , based on the available bus voltage.

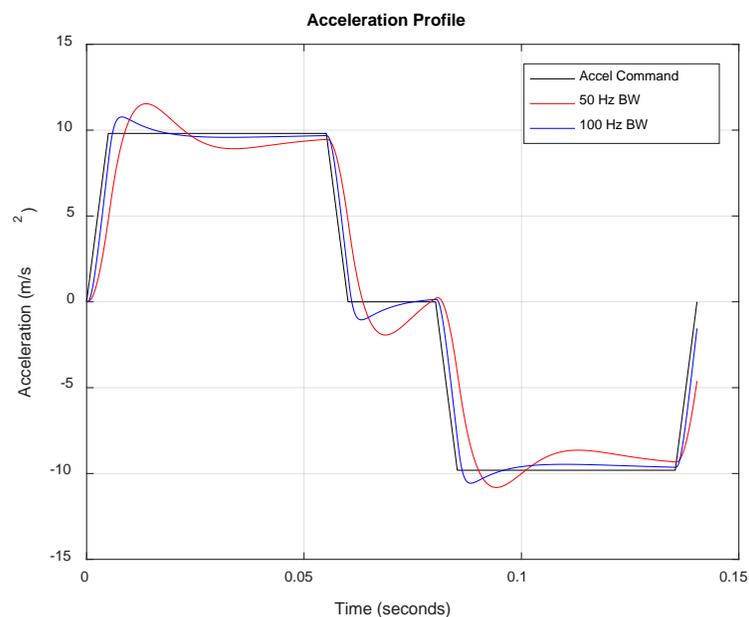


Figure 2 Motion Trajectory (Black - Commanded, Red - 50 Hz, Green - 100 Hz)

## CASE 2 - DISTURBANCE TRACKING

In cases where the control system is reacting to broadband energy (trying to hold focus, position or velocity), the servo bandwidth may be limited by motor constant  $K_m$ . As the units of the motor constant state  $(\frac{N-m}{\sqrt{W}})$ ,  $K_m$  is a measure a motor's ability to make torque per the square root of dissipated power.

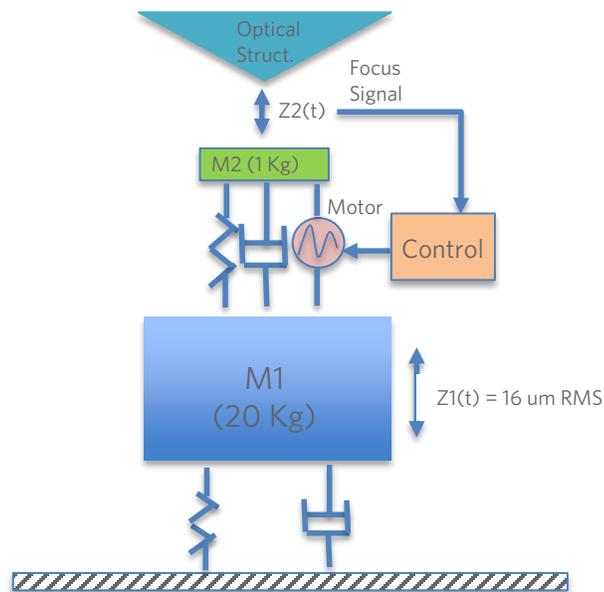


Figure 3 Simple Disturbance Tracking System

Figure 3 above depicts a system where the job of the control system is to minimize focus error between the optical structure and M2, while M1 is subjected to a disturbance force, creating motion  $Z_1(t)$ . In our example, the input spectrum  $Z_1(t)$  has broadband energy up to the kHz frequency range.

Figure 4 below shows the plant and loop transfer functions for the system. The loop is showing ~50 Hz 0 dB crossing. We will call this frequency the "servo bandwidth". Figure 5 is a plot of motor force for the 50 Hz case and also the 100 Hz 0 dB case, showing how the bandwidth dramatically effects the amplitude of the motor force signal. Dissipated power in the system is;

$$P_d = \frac{\text{Duty Cycle\%} * F_{rms}^2}{K_m}$$

where  $K_m$  is the motor constant in  $(\frac{N}{\sqrt{W}})$ , and Duty Cycle is 100%.  $K_m$  is calculated using;

$$K_m = \frac{2}{\sqrt{3}} \frac{K_f}{\sqrt{2 * R_{ph}}}$$

In our example,  $K_m$  is calculated to be 29 N/sqrt(W, where  $K_f = 50$  N/amp, and  $R_{ph} = 2$  ohms. The dissipated power for these two cases are 12 watts for the 50 Hz case and 75 watts for the 100 Hz case. Clearly, the higher bandwidth case requires more power (6x increase) to track the higher frequencies.

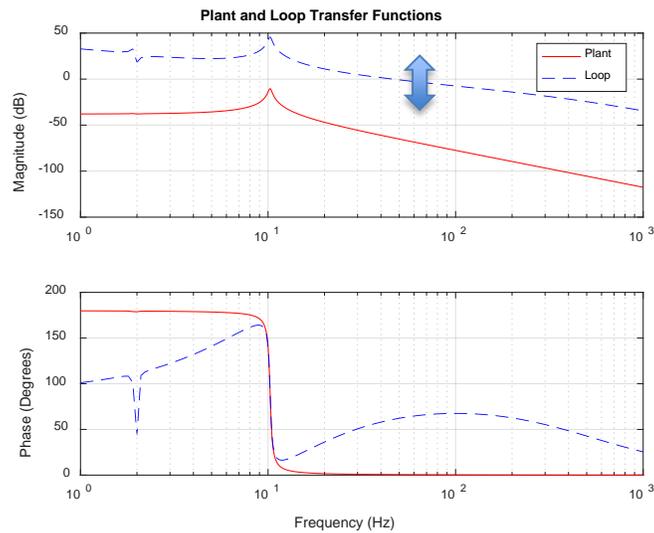


Figure 4. Plant and Loop Transfer Functions

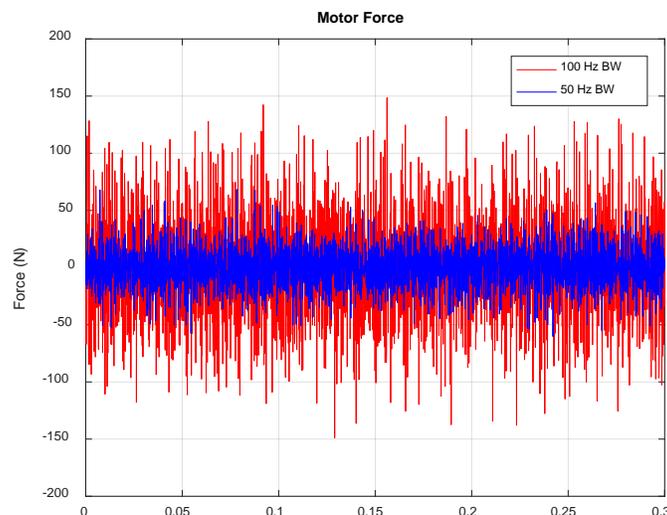


Figure 5. Motor force for 50 Hz vs. 100 Hz Bandwidth

The example given above is summarized in Figure 6. The net result is that higher bandwidth systems track higher frequency disturbances. Higher frequency disturbances typically mean higher accelerations and thus higher required motor forces. Power dissipation goes with the square of motor force, so any increase in bandwidth frequency can have a significant impact on motor dissipated power and motor sizing.

In Figure 6 below, the final plot is the resulting acceleration for three cases of servo bandwidths. The yellow is for the 100 Hz case, green for the 75 case, and red for the 50 Hz case. As the pictorials show, the RMS of acceleration, and resulting force, is the area under the acceleration spectrum plot, and the smaller the bandwidth is, the smaller the RMS acceleration will be.

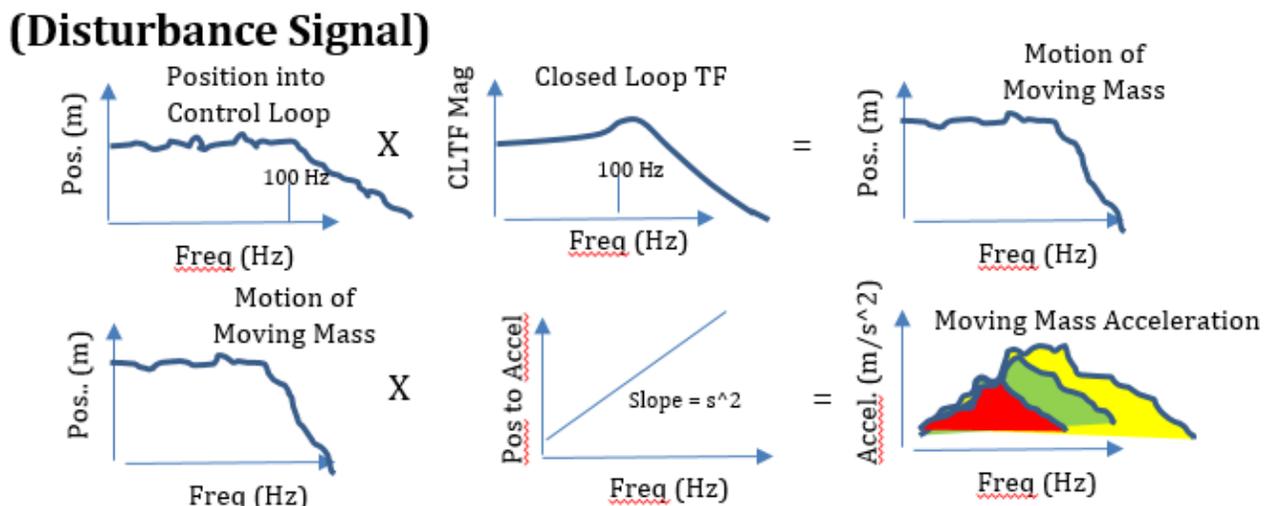


Figure 6. Pictorial of Analysis

## CASE STUDY SUMMARIES

Described here are two very common, yet different, cases that affect motor sizing. Case 1 is when the control system has to move a load from position A to position B and repeat. Case 2 is where the control system has to track (or reject) a disturbance signal. In each case power dissipated in the motor is the ultimate driving factor.

In case 1, the dissipated power is primarily a function of the duty cycle of motion, the moving mass (or inertia), and the acceleration reached during the move and is independent of servo loop bandwidth. In

this case, the “backbone” of the motion trajectory is the same independently of servo bandwidth (within the amount of servo tracking error).

In case 2, however, dissipated power is largely driven by the servo bandwidth. Higher bandwidth servos ask the controller to track higher frequencies, which means higher accelerations and higher motor forces. In our example, a 2x increase in servo bandwidth created 6x more dissipated power in the motor.

In both cases, servo bandwidth is ultimately governed by servo stability, meaning gain margin and phase margin must always be met. In some cases like case 2 described here, the servo bandwidth might not be limited by stability, but by allowable dissipated power.

In both cases, dissipated power is used to size the motor. The load mass (or inertia), its acceleration, and its duty cycle all combine to affect motor power dissipation.

## CONCLUSIONS

Motor sizing is driven by allowable power dissipation in the motor. When choosing a motor, one must consider how it will be used and do the appropriate analysis to arrive at the predicted motor power dissipation. Described here are two cases that require two different types of analysis. In case 1, the analysis is focused on the motion trajectory, while in case 2, the analysis is focused on the servo bandwidth. In both cases, the acceleration of the load, and its mass drive motor power dissipation.